'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2023

GE1-P2-MATHEMATICS

(REVISED SYLLABUS 2023)

Time Allotted: 2 Hours — Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains MATHGE-II, MATHGE-III & MATHGE-V.
The candidates are required to answer any *one* from the *three* courses.
Candidates should mention it clearly on the Answer Book.

MATHGE-II

ALGEBRA

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4 = 12$

(a) If α be a multiple root of order 3 of the equation $x^4+bx^3+cx+d=0\ (d\neq 0)$, then show that $\alpha=\dfrac{-8d}{3c}$. 3

(b) Applying Descarte's rule of signs, find the nature of the roots of 3

$$x^6+x^4+x^2+2x+5=0$$

(c) Find the product of all values of $(1+i)^{4/5}$. 3

(d) If a, b, c be three positive real numbers, then show that $\left(\dfrac{a+b+c}{3}\right)^3\geq a\left(\dfrac{b+c}{2}\right)^2$. 3

(e) Verify Cayley-Hamilton theorem for the square matrix $\begin{pmatrix}2 & 1\\ 0 & 5\end{pmatrix}$. 3

(f) Express $\dfrac{1+i\sqrt{3}}{1-i}$ in the polar form and hence find the value of $\sin\dfrac{5\pi}{12}$. 3

GROUP-B

Answer any *four* questions from the following $6\times4 = 24$

2. If $x=\log\tan\left(\dfrac{\pi}{4}+\dfrac{y}{2}\right)$, then prove that $y=-i\log\tan\left(\dfrac{\pi}{4}+i\dfrac{x}{2}\right)$. 6

3. Reduce the equation $x^3-3x^2+12x+16=0$ to its standard form and then solve the equation by Cardon's method. 6

4. Let M be an 3×3 real matrix with eigen values 2, 3, 1 and the corresponding eigen vectors $(1, 2, 1)^t$, $(0, 1, 1)^t$, $(1, 1, 1)^t$ respectively. Determine the matrix M. 6

5. Let a, b, c, d be positive real numbers not all equal. Show that 6

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)>16$$

6. Determine the conditions for which the following system of equations — 6

$$x+y+z=b;$$
$$2x+y+3z=b+1;$$
$$5x+2y+9z=b^2$$

has (i) only one solution, (ii) no solution, (iii) many solutions.

7. Let S be a set containing n elements, where n is a positive integer. If r is an integer such that $0 \le r \le n$, then show that the numbers of subsets of S containing exactly r elements is $\dfrac{n!}{r!(n-r)!}$. — 6

GROUP-C

Answer any *two* questions from the following — 12×2 = 24

8. (a) State and prove division algorithm. — 6

(b) State the well ordering principle. Show that $2^{2n+1}-9n^2+3n-2$ is divisible by 54. — 1+5

9. (a) If $\alpha, \beta, \gamma, \delta$ be the roots of $x^4-3x^3+4x^2-5x+6=0$, show that the value of $(\alpha^2+3)(\beta^2+3)(\gamma^2+3)(\delta^2+3)$ is 57. — 6

(b) If $a_1, a_2, a_3, \cdots\cdots, a_n$ be n positive real numbers, then show that $\dfrac{s}{s-a_1}+\dfrac{s}{s-a_2}+\dfrac{s}{s-a_3}+\cdots\cdots+\dfrac{s}{s-a_n} \ge \dfrac{n^2}{n-1}$, where $s=a_1+a_2+a_3+\cdots\cdots+a_n$. — 6

10.(a) Show that the relation $a \equiv b \pmod 5$ is an equivalence relation. — 6

(b) If $\log\sin(\theta+i\phi)=\alpha+i\beta$, then prove that $2\cos 2\theta = e^{2\phi}+e^{-2\phi}-4e^{2\alpha}$ and $\cos(\theta-\beta)=e^{2\phi}\cos(\theta+\beta)$. — 6

11.(a) Find the eigen values and the corresponding eigen vectors of the matrix — 6

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 2 & 0 \end{pmatrix}$$

Further find the algebraic and geometric multiplicities of the eigen values.

(b) If $A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then show that for every integer $n(\ge 3)$, $A^n = A^{n-2}+A^2-I$ — 4+2

Hence find A^{50}.

MATHGE-III

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

GROUP-A

Answer any *four* questions — 3×4 = 12

1. Show that the functions $\{e^x, e^{2x}, e^{3x}\}$ are linearly independent solutions of the differential equation $\dfrac{d^3y}{dx^3}-6\dfrac{d^2y}{dx^2}+11\dfrac{dy}{dx}-6y=e^{2x}$.

2. Solve: $\dfrac{d^4y}{dx^4}+m^4y=0$

3. For what value of k, the straight lines $\vec{r}=(1,\,2,\,3)+t(2,\,3,\,4)$ and $\vec{r}=(k,\,3,\,4)+s(3,\,4,\,5)$ (where t, s are scalars) are coplanar.

4. Evaluate $\int_1^2 \vec{r}\times\dfrac{d^2\vec{r}}{dt^2}\,dt$, where $\vec{r}=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$.

5. Solve the initial value problem $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y=0$, $y=\dfrac{dy}{dx}=3$ at $x=0$.

6. Find particular integral of $(D^3-D^2+3D+5)y=e^x\cos x$.

GROUP-B

Answer any *four* questions $6\times4=24$

7. Given a vector field $\vec{V}=x\hat{i}+y\hat{j}+z\hat{k}$ in E^3. Find curl$\left(\dfrac{\vec{V}}{|\vec{V}|}\right)$.

8. Let $\vec{F}=xy\hat{i}+(x^2+y^2)\hat{j}$. Then obtain $\int_\Gamma \vec{F}\cdot d\vec{r}$, where Γ is the arc of the parabola $y=x^2-4$ from (2, 0) to (4, 12).

9. Solve $(D^2+3D+2)y=x+\cos x$ by method of undetermined co-efficient.

10. Solve the differential equation $\dfrac{d^2x}{dt^2}-\mu x=0$ with the condition $x=a$, $\dfrac{dx}{dt}=-V$ when $t=0$.

11. Solve the system of differential equation $\dfrac{dx}{dt}+2x-3y=t$, $-3x+\dfrac{dy}{dt}+2y=e^{2t}$.

12. Solve $(D^4-4D^2-5)y=e^x(x+\cos x)$ using D operator.

GROUP-C

Answer any *two* questions $12\times2=24$

13.(a) Solve: $(D^3-5D^2+7D-3)y=e^{2x}\cosh x$ 6+6

(b) Solve: $\dfrac{dx}{dt}-\dfrac{dy}{dt}+3x=\sin t$; $\dfrac{dx}{dt}+y=\cos t$, given that $x=1$, $y=0$ at $t=0$.

14.(a) Find the series solution of $4x\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y=0$. 8+4

(b) If $\vec{r}$ be a position vector of a point and $\phi=\dfrac{1}{|\vec{r}|}$, then show that $\nabla\phi=-\dfrac{\vec{r}}{|\vec{r}|^3}$.

15.(a) If $F=\phi\,\text{grad}\,\phi$, then show that $F\cdot\text{curl}\,F=0$. 6+6

(b) If $\vec{F}=(3x^2+6y)\hat{i}-14yz\hat{j}+20zx^3\hat{k}$, then evaluate $\int_C \vec{F}\cdot d\vec{r}$, from (0, 0, 0) to (1, 1, 1) along the curve $x=t$, $y=t^2$, $z=t^3$.

16.(a) Solve $(x^2D^2 - xD + 4)y = \cos(\log x) + x\sin(\log x)$. 6+6

(b) Solve the following differential equation using method of variation of parameter

$$(D^2 - 2D + 2)y = e^x \tan x$$

MATHGE-V

NUMERICAL METHODS

GROUP-A

1. Answer any ***four*** questions from the following: $3\times4 = 12$

(a) Prove that $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \nabla + \Delta$, where Δ and ∇ have their usual meaning.

(b) Define the degree of precision of a quadrature formula for numerical integration. What is the degree of precision of Simpson's $\frac{1}{3}^{\text{rd}}$ rule?

(c) Find the relative error in computation of $x + y$ for $x = 11.75$ and $y = 7.23$ having absolute errors $\Delta x = 0.002$ and $\Delta y = 0.005$ respectively.

(d) What is the geometric representation of the Trapezoidal rule for integrating $\int_a^b f(x)\,dx$?

(e) If $h = 1$ then find the value of $\Delta^3(1-x)(1-2x)(1-3x)$.

(f) Write down the equation $x^3 + 2x - 10 = 0$ in the form $x = \phi(x)$ such that the iterative scheme about $x = 2$ converges.

GROUP-B

Answer any *four* questions from the following $6\times4 = 24$

2. If a number be rounded to n correct significant figures, then prove that relative error is less than $\frac{1}{k\times10^{n-1}}$.

3. The third order differences of a function $f(x)$ are constant and $\int_{-1}^{1} f(x)\,dx = k\left[f(0) + f\left(\frac{1}{\sqrt{2}}\right) + f\left(-\frac{1}{\sqrt{2}}\right)\right]$, then find the value of k.

4. Using Regula-Falsi Method, find a root of $x^3 + 2x + 2 = 0$, correct up to three significant figures.

5. Solve the following differential equation for $x = 0.02$ by taking step length $h = 0.01$ by modified Euler's method:

$$\frac{dy}{dx} = x^2 + y,\ y = 1 \text{ when } x = 0$$

6. Establish Newton's backward interpolation formula.

7. Solve the system by Gauss-Seidel iteration method:

$$x + y + 4z = 9$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Estimate the missing term in the following table: 3

x	0	1	2	3	4
$f(x)$	1	3	9	-	81

(b) Explain the geometrical interpretation of Simpson's $\frac{1}{3}^{\text{rd}}$ rule. 3

(c) Show that number of multiplications and divisions for the linear system of n equations having n unknowns by elimination method is about $n^3/3$. 6

9. (a) Evaluate $f(x)$ for $x = 0.07$ using the given values: 6

x	0.00	0.10	0.20	0.30	0.40
$f(x)$	1.0000	1.2214	1.4918	1.8221	2.2255

(b) Using Runge-Kutta method of order 2 to calculate $y(0.6)$ for the equation $\frac{dy}{dx} = x + y^2$, $y(0) = 1$ taking $h = 0.2$. 6

10.(a) Deduce an expression for the remainder in polynomial interpolation of a function $f(x)$ with nodes $x_0, x_1, x_2, \cdots\cdots, x_n$. 6

(b) Find the value $\int_1^5 \log_{10} x\, dx$ taking 8 sub-intervals, correct up to four decimal places by Trapezoidal rule. 6

11.(a) Compute the values of the unknowns in the system of equations by Gauss Jordan's matrix inversion method 6

$$3x + 4y - 2z = 15$$
$$5x + 2y + z = 18$$
$$2x + 3y + 3z = 10$$

(b) Find the value of $\int_0^1 \frac{dx}{1+x^2}$ taking 5-sub-intervals by Simpson's $\frac{3}{8}^{\text{th}}$ rule, correct up to four significant figures. 6

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UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2023

GE1-P2-MATHEMATICS

(OLD SYLLABUS 2018)

Time Allotted: 2 Hours — Full Marks: 60

The figures in the margin indicate full marks.

The question paper contains MATHGE-I, MATHGE-II, MATHGE-III, MATHGE-IV & MATHGE-V.
The candidates are required to answer any *one* from the *five* courses.
Candidates should mention it clearly on the Answer Book.

MATHGE-I

CAL. GEO. AND DE.

GROUP-A

1. Answer any ***four*** questions from the following: $3\times 4 = 12$

(a) If the graph of $f(x)=\dfrac{x^2-3x+4}{cx^2-x-10}$ has horizontal asymptote at $y=\dfrac{1}{2}$, find c. 3

(b) Find $\lim\limits_{x\to 0}\dfrac{x^2\sin(1/x)}{\sin x}$. 3

(c) Show that the conic $x^2+2xy+y^2-2x-1=0$ is parabola. 3

(d) Show that origin is a point of inflexion on the curve $y=x\cos 2x$. 3

(e) When the axes are turned through an angle, the expression $ax+by$ becomes $a'x'+b'y'$ referred to new ones. Show that $a+b=a'+b'$. 3

(f) Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle. 3

GROUP-B

Answer any *four* questions from the following $6\times 4 = 24$

2. Trace the curve $x(x^2+y^2)=a(x^2-y^2)$, $a>0$. 6

3. Show that the straight line $\dfrac{l}{r}=A\cos\theta+B\sin\theta$ touches the conic $\dfrac{l}{r}=1+e\cos\theta$, if $(A-e)^2+B^2=1$. 6

4. If $y=\sin^{-1}x$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. 6

5. Solve: $(x^2+y^2+2x)\,dx+2y\,dy=0$ 6

6. Solve: $x^3\dfrac{dy}{dx}-x^2y+y^4\cos x=0$ 6

7. Find the envelope of family of co-axial ellipses $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, where the parameters are connected by the relation $a+b=c$ (c being fixed). 6

GROUP-C

Answer any *two* questions from the following $12\times 2=24$

8. (a) Find the equation of the cylinder whose generating line is parallel to the z axis and the guiding curve is $x^2+y^2=z,\ x+y+z=1$. 6

(b) Find the reduction formula for $\int \sin^n x\,dx$ and hence find $\int_0^{\pi/2}\sin^5 x\,dx$. 6

9. (a) Find the values of a, b and c for which $\lim\limits_{x\to 0}\dfrac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$. 6

(b) If a plane passes through a fixed point (α,β,γ) and cuts the axes is P, Q, R. Show that the locus of the centre of the sphere passing through the origin and points P, Q, R is $\dfrac{\alpha}{x}+\dfrac{\beta}{y}+\dfrac{\gamma}{z}=2$. 6

10.(a) Solve: $\dfrac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ 6

(b) Solve: $(x+1)\dfrac{dy}{dx}-ny=e^x(x+1)^{n+1}$ 6

11.(a) Discuss the nature of the surface $4x^2-y^2-z^2+2yz-8x-4y+8z-2=0$. Reduce it to its canonical form. 6

(b) Find the area of the surface obtained by revolving the parametric curve defined by $x(t)=3t-t^3,\ y(t)=3t^2,\ 0\le t\le 1$ about the x-axis. 3

(c) Prove that the area bounded between one arch of the cycloid $x=a(t-\sin t),\ y=a(1-\cos t)$ and the x-axis is $3\pi a^2$. 3

MATHGE-II

ALGEBRA

GROUP-A

1. Answer any ***four*** questions from the following: $3\times 4=12$

(a) If α be a multiple root of order 3 of the equation $x^4+bx^3+cx+d=0\ (d\neq 0)$, then show that $\alpha=\dfrac{-8d}{3c}$. 3

(b) Applying Descarte's rule of signs, find the nature of the roots of 3

$$x^6+x^4+x^2+2x+5=0$$

(c) Find the product of all values of $(1+i)^{4/5}$. 3

(d) If a, b, c be three positive real numbers, then show that $\left(\dfrac{a+b+c}{3}\right)^3\ge a\left(\dfrac{b+c}{2}\right)^2$. 3

(e) Verify Cayley-Hamilton theorem for the square matrix $\begin{pmatrix}2 & 1\\ 0 & 5\end{pmatrix}$. 3

(f) Express $\dfrac{1+i\sqrt{3}}{1-i}$ in the polar form and hence find the value of $\sin\dfrac{5\pi}{12}$. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. If $x=\log\tan\left(\frac{\pi}{4}+\frac{y}{2}\right)$, then prove that $y=-i\log\tan\left(\frac{\pi}{4}+i\frac{x}{2}\right)$. 6

3. Reduce the equation $x^3-3x^2+12x+16=0$ to its standard form and then solve the equation by Cardon's method. 6

4. Let M be an 3×3 real matrix with eigen values 2, 3, 1 and the corresponding eigen vectors $(1,2,1)^t$, $(0,1,1)^t$, $(1,1,1)^t$ respectively. Determine the matrix M. 6

5. Let a, b, c, d be positive real numbers not all equal. Show that 6

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)>16$$

6. Determine the conditions for which the following system of equations 6

$$x+y+z=b;$$
$$2x+y+3z=b+1;$$
$$5x+2y+9z=b^2$$

has (i) only one solution, (ii) no solution, (iii) many solutions.

7. Let S be a set containing n elements, where n is a positive integer. If r is an integer such that $0\le r\le n$, then show that the numbers of subsets of S containing exactly r elements is $\frac{n!}{r!(n-r)!}$. 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) State and prove division algorithm. 6

(b) State the well ordering principle. Show that $2^{2n+1}-9n^2+3n-2$ is divisible by 54. 1+5

9. (a) If $\alpha, \beta, \gamma, \delta$ be the roots of $x^4-3x^3+4x^2-5x+6=0$, show that the value of $(\alpha^2+3)(\beta^2+3)(\gamma^2+3)(\delta^2+3)$ is 57. 6

(b) If $a_1, a_2, a_3, \cdots\cdots, a_n$ be n positive real numbers, then show that 6

$$\frac{s}{s-a_1}+\frac{s}{s-a_2}+\frac{s}{s-a_3}+\cdots\cdots+\frac{s}{s-a_n}\ge\frac{n^2}{n-1},\text{ where } s=a_1+a_2+a_3+\cdots\cdots+a_n.$$

10.(a) Show that the relation $a\equiv b\ (\text{mod } 5)$ is an equivalence relation. 6

(b) If $\log\sin(\theta+i\phi)=\alpha+i\beta$, then prove that $2\cos 2\theta=e^{2\phi}+e^{-2\phi}-4e^{2\alpha}$ and $\cos(\theta-\beta)=e^{2\phi}\cos(\theta+\beta)$. 6

11.(a) Find the eigen values and the corresponding eigen vectors of the matrix 6

$$\begin{pmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & 2 & 0\end{pmatrix}$$

Further find the algebraic and geometric multiplicities of the eigen values.

(b) If $A=\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$, then show that for every integer $n(\ge 3)$, $A^n=A^{n-2}+A^2-I$ 4+2

Hence find A^{50}.

MATHGE-III

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

GROUP-A

Answer any *four* questions 3×4 = 12

1. Show that the functions $\{e^x, e^{2x}, e^{3x}\}$ are linearly independent solutions of the differential equation $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+11\frac{dy}{dx}-6y=e^{2x}$.

2. Solve: $\frac{d^4y}{dx^4}+m^4y=0$

3. For what value of k, the straight lines $\vec{r}=(1, 2, 3)+t(2, 3, 4)$ and $\vec{r}=(k, 3, 4)+s(3, 4, 5)$ (where t, s are scalars) are coplanar.

4. Evaluate $\int_1^2 \vec{r}\times\frac{d^2\vec{r}}{dt^2}\,dt$, where $\vec{r}=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$.

5. Solve the initial value problem $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$, $y=\frac{dy}{dx}=3$ at $x=0$.

6. Find particular integral of $(D^3-D^2+3D+5)y=e^x\cos x$.

GROUP-B

Answer any *four* questions 6×4 = 24

7. Given a vector field $\vec{V}=x\hat{i}+y\hat{j}+z\hat{k}$ in E^3. Find curl$\left(\frac{\vec{V}}{|\vec{V}|}\right)$.

8. Let $\vec{F}=xy\hat{i}+(x^2+y^2)\hat{j}$. Then obtain $\int_\Gamma \vec{F}\cdot d\vec{r}$, where Γ is the arc of the parabola $y=x^2-4$ from (2, 0) to (4, 12).

9. Solve $(D^2+3D+2)y=x+\cos x$ by method of undetermined co-efficient.

10. Solve the differential equation $\frac{d^2x}{dt^2}-\mu x=0$ with the condition $x=a$, $\frac{dx}{dt}=-V$ when $t=0$.

11. Solve the system of differential equation $\frac{dx}{dt}+2x-3y=t$, $-3x+\frac{dy}{dt}+2y=e^{2t}$.

12. Solve $(D^4-4D^2-5)y=e^x(x+\cos x)$ using D operator.

GROUP-C

Answer any *two* questions 12×2 = 24

13.(a) Solve: $(D^3-5D^2+7D-3)y=e^{2x}\cosh x$ 6+6

(b) Solve: $\frac{dx}{dt}-\frac{dy}{dt}+3x=\sin t$; $\frac{dx}{dt}+y=\cos t$, given that $x=1$, $y=0$ at $t=0$.

14.(a) Find the series solution of $4x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$. 8+4

(b) If $\vec{r}$ be a position vector of a point and $\phi=\frac{1}{|\vec{r}|}$, then show that $\nabla\phi=-\frac{\vec{r}}{|\vec{r}|^3}$.

15.(a) If $F = \phi\,\text{grad}\phi$, then show that $F \cdot \text{curl}\, F = 0$. 6+6

(b) If $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20zx^3\hat{k}$, then evaluate $\int_C \vec{F} \cdot d\vec{r}$, from (0, 0, 0) to (1, 1, 1) along the curve $x = t,\ y = t^2,\ z = t^3$.

16.(a) Solve $(x^2D^2 - xD + 4)y = \cos(\log x) + x\sin(\log x)$. 6+6

(b) Solve the following differential equation using method of variation of parameter

$$(D^2 - 2D + 2)y = e^x \tan x$$

MATHGE-IV

Group Theory

GROUP-A

Answer any *four* questions from the following 3×4 = 12

1. Let $H_1,\ H_2$ be two subgroups of a group *G*. Prove that $H_1 \cap H_2$ is also a subgroup of *G*. 3
2. Find all generators of the group $(\mathbb{Z}_8, +_8)$. 3
3. Show that a group of even order has an element of order 2 and that the number of elements of order 2 is odd. 3
4. If *H* and *K* are two normal subgroups of a group *G* such that $H \cap K = \{e\}$, then show that $hk = kh\ \ \forall\, h \in H,\, k \in K$. 3
5. Let $\mathbb{R}^+$ be the group of positive real numbers under multiplication and $\mathbb{R}$ the group of all real numbers under addition. Then show that the map $\theta :\ \mathbb{R}^+ \longrightarrow \mathbb{R}$ such that $\theta(x) = \log_e x$ is an isomorphism. 3
6. If *G* and G' be two groups and $f : G \to G'$ be a homomorphism then show that $f(e) = e'$. 3

GROUP-B

Answer any *four* questions from the following 6×4 = 24

7. Let *a* be an element of a finite group *G*. Prove that $O(a) \mid O(G)$. 6
8. Let *H* be a subgroup of a group *G*. Then prove that $Ha = Hb$ if and only if $ab^{-1} \in H$, where $a, b \in G$. 6
9. Define centre of a group. Prove that the centre of a group *G* is a subgroup of *G*. 6
10. Prove that the set *H* forms a commutative group with respect to matrix multiplication, where $H = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} :\ a \in \mathbb{R} \ \text{ and } a^2 + b^2 = 1 \right\}$. 6
11. Prove that a non-empty subset *H* of a group *G* is a subgroup of *G* if and only if $a, b \in H \Rightarrow ab^{-1} \in H$. 6
12. Prove that the order of a cyclic group is equal to the order of its generator. 6

GROUP-C

Answer any *two* questions from the following 12×2 = 24

13.(a) Prove that every group *G* is isomorphic to a permutation group. 7

(b) Let $f : G \to G'$ be a group homomorphism. Let $a \in G$ such that $O(a) = n$ and 5

$O(f(a)) = m$. Show that $O(f(a)) \mid O(a)$ and f is one-one iff $m = n$.

14.(a) If H and K be subgroups of a group G, then show that $O(HK) = \dfrac{O(H)\,O(K)}{O(H \cap K)}$. 7

(b) If $f: G \to G'$ be a group homomorphism, then show that $\ker f$ is a normal subgroup of G. 5

15.(a) Prove that every quotient group of a cyclic group is cyclic. 6

(b) Prove that a group homomorphism $f: G \to G'$ is one-one iff $\ker f = \{e\}$. 6

16.(a) Find all the group homomorphisms from $\mathbb{Z}_{20} \longrightarrow \mathbb{Z}_8$. How many of these are onto? 6

(b) (i) Prove that a group G is commutative iff $(ab)^{-1} = a^{-1}b^{-1}$, $\forall\ a, b \in G$. 4+2

(ii) Examine whether $(\mathbb{Z}, \circ)$ forms a group with respect to the composition '$\circ$' by $a \circ b = a + b + ab$; $a, b \in \mathbb{Z}$.

MATHGE-V

Numerical Methods

GROUP-A

1. Answer any ***four*** questions from the following: 3×4 = 12

(a) Prove that $\dfrac{\Delta}{\nabla} - \dfrac{\nabla}{\Delta} = \nabla + \Delta$, where Δ and ∇ have their usual meaning.

(b) Define the degree of precision of a quadrature formula for numerical integration. What is the degree of precision of Simpson's $\frac{1}{3}^{\text{rd}}$ rule?

(c) Find the relative error in computation of $x + y$ for $x = 11.75$ and $y = 7.23$ having absolute errors $\Delta x = 0.002$ and $\Delta y = 0.005$ respectively.

(d) What is the geometric representation of the Trapezoidal rule for integrating $\int_a^b f(x)\,dx$?

(e) If $h = 1$ then find the value of $\Delta^3 (1-x)(1-2x)(1-3x)$.

(f) Write down the equation $x^3 + 2x - 10 = 0$ in the form $x = \phi(x)$ such that the iterative scheme about $x = 2$ converges.

GROUP-B

Answer any *four* questions from the following 6×4 = 24

2. If a number be rounded to n correct significant figures, then prove that relative error is less than $\dfrac{1}{k \times 10^{n-1}}$.

3. The third order differences of a function $f(x)$ are constant and $\int_{-1}^{1} f(x)\,dx = k\left[f(0) + f\left(\frac{1}{\sqrt{2}}\right) + f\left(-\frac{1}{\sqrt{2}}\right)\right]$, then find the value of k.

4. Using Regula-Falsi Method, find a root of $x^3 + 2x + 2 = 0$, correct up to three significant figures.

5. Solve the following differential equation for $x = 0.02$ by taking step length

$h = 0.01$ by modified Euler's method:

$$\frac{dy}{dx} = x^2 + y,\ \ y = 1 \text{ when } x = 0$$

6. Establish Newton's backward interpolation formula.

7. Solve the system by Gauss-Seidel iteration method:

$$x + y + 4z = 9$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

GROUP-C

Answer any *two* questions from the following 12×2 = 24

8. (a) Estimate the missing term in the following table: 3

x	0	1	2	3	4
$f(x)$	1	3	9	-	81

(b) Explain the geometrical interpretation of Simpson's $\frac{1}{3}^{\text{rd}}$ rule. 3

(c) Show that number of multiplications and divisions for the linear system of n equations having n unknowns by elimination method is about $n^3/3$. 6

9. (a) Evaluate $f(x)$ for $x = 0.07$ using the given values: 6

x	0.00	0.10	0.20	0.30	0.40
$f(x)$	1.0000	1.2214	1.4918	1.8221	2.2255

(b) Using Runge-Kutta method of order 2 to calculate $y(0.6)$ for the equation $\frac{dy}{dx} = x + y^2,\ y(0) = 1$ taking $h = 0.2$. 6

10.(a) Deduce an expression for the remainder in polynomial interpolation of a function $f(x)$ with nodes $x_0, x_1, x_2, \cdots\cdots, x_n$. 6

(b) Find the value $\int_1^5 \log_{10} x\, dx$ taking 8 sub-intervals, correct up to four decimal places by Trapezoidal rule. 6

11.(a) Compute the values of the unknowns in the system of equations by Gauss Jordan's matrix inversion method 6

$$3x + 4y - 2z = 15$$
$$5x + 2y + z = 18$$
$$2x + 3y + 3z = 10$$

(b) Find the value of $\int_0^1 \frac{dx}{1+x^2}$ taking 5-sub-intervals by Simpson's $\frac{3}{8}^{\text{th}}$ rule, correct up to four significant figures. 6

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